Week 4

Grammars Pt. 2

Anakin

Outline

Context Free Grammars

Pushdown Automata

Section 1

Context Free Grammars

Quick Review

$\left\{ \mathsf{0}^n\mathsf{1}^n \;\middle|\; n \geq 0 \right\}$ is generated by

Quick Review

$\left\{ \mathtt{0}^n\mathtt{1}^n \;\middle|\; n \geq 0 \right\}$ is generated by

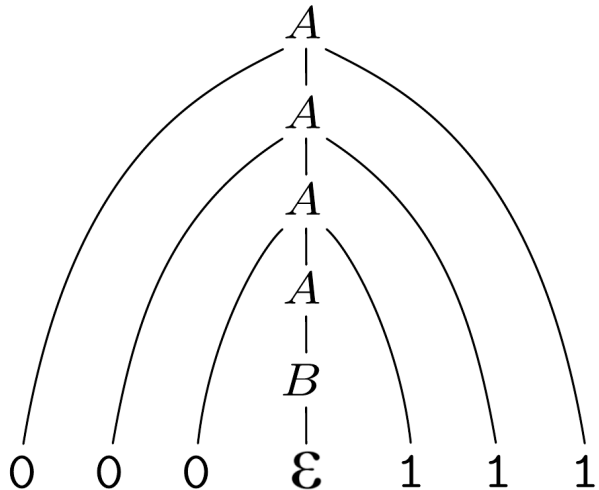
$$\begin{aligned} A &\rightarrow \mathtt{0}A\mathtt{1} \\ A &\rightarrow B \\ B &\rightarrow \varepsilon \end{aligned}$$

Quick Review

$\left\{ \mathtt{0}^n \mathtt{1}^n \;\middle|\; n \geq 0 \right\}$ is generated by

$$A \;\rightarrow\; \mathtt{0}A\mathtt{1} \mid \varepsilon$$

Determining If Strings Are In A CFL



Determining If Strings Are In A CFL

$$A \Rightarrow 0A1 \Rightarrow 00A11 \Rightarrow 000A111 \Rightarrow 000B111 \Rightarrow 000\varepsilon111 \Rightarrow 000111$$

Comparison with Regular Languages

- CFGs define a language much like Regex/DFAs/NFAs

Comparison with Regular Languages

- CFGs define a language much like Regex/DFAs/NFAs
- Regex / DFAs / NFAs $\leftrightarrow$ Regular Languages
- Context Free Grammars $\leftrightarrow$ Context Free Languages

Questions!

This is going to be a review of a ton of things we've talked about since automata are coming back

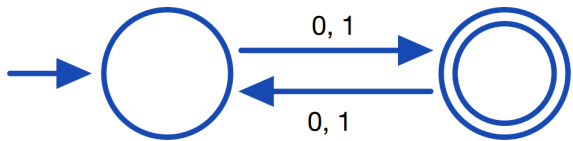
Questions!

This is going to be a review of a ton of things we've talked about since automata are coming back

- Create a CFG and NFA / DFA to recognize
  $\{ w \mid \text{the length of } w \text{ is odd} \}$
- Create a CFG to recognize
  $\{ w \mid \text{the length of } w \text{ is odd and the middle character is } \texttt{0} \}$

Answers

Create a CFG and NFA / DFA to recognize
$\{ w \mid \text{the length of } w \text{ is odd} \}$



$$S \rightarrow 1E \mid 0E$$
$$E \rightarrow \varepsilon \mid 0S \mid 1S$$

Answers

Create a CFG to recognize

$\{ w \mid$ the length of w is odd and the middle character is $\texttt{0} \}$

$$
\begin{aligned}
S &\rightarrow \texttt{0} \mid TST \\
T &\rightarrow \texttt{0} \mid \texttt{1}
\end{aligned}
$$

Section 2

Pushdown Automata

Automata for CFLs

- Regex $\leftrightarrow$ DFAs / NFAs

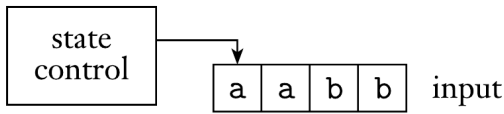
Automata for CFLs

- Regex $\leftrightarrow$ DFAs / NFAs
- Context Free Grammars $\leftrightarrow$ **???**

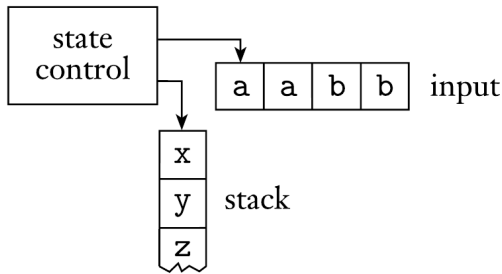
Automata for CFLs

- Regex $\leftrightarrow$ DFAs / NFAs
- Context Free Grammars $\leftrightarrow$ **Pushdown Automata**

State Machines With an Upgrade



State Machines With an Upgrade



What is a Stack??

- Think about stacking objects (books, plates, whatever)

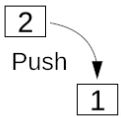
What is a Stack??

- Think about stacking objects (books, plates, whatever)
- You can add items to the **top only** and lose immediate access to anything below it

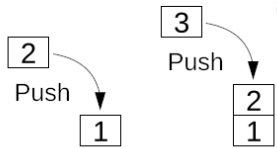
What is a Stack??

- Think about stacking objects (books, plates, whatever)
- You can add items to the **top only** and lose immediate access to anything below it
- If you want to get an item from your stack, you have to pick up the **top item first** and then discard it

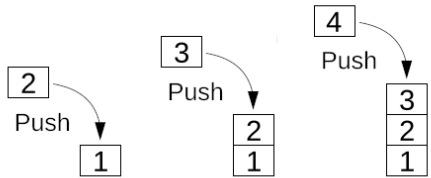
Stack



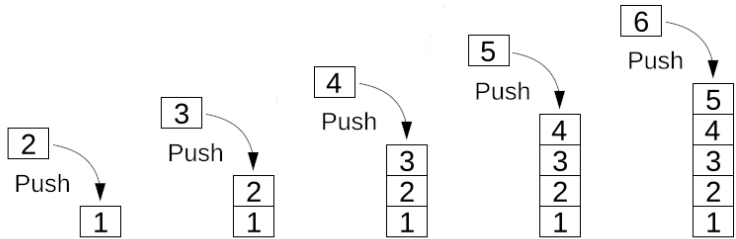
Stack



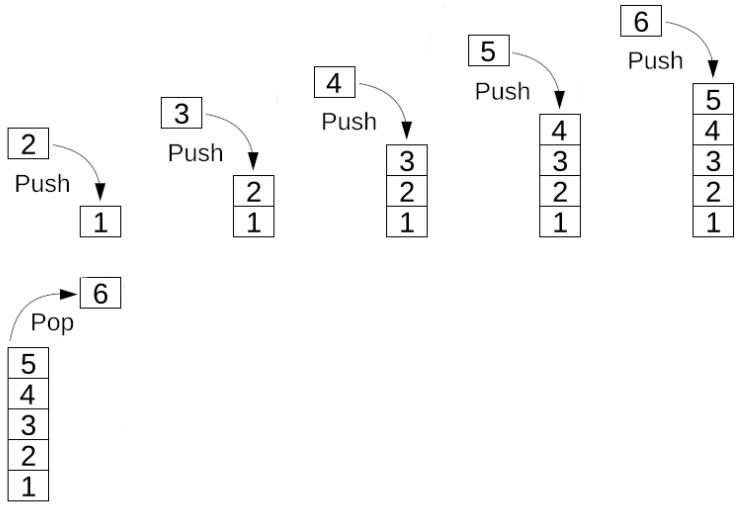
Stack



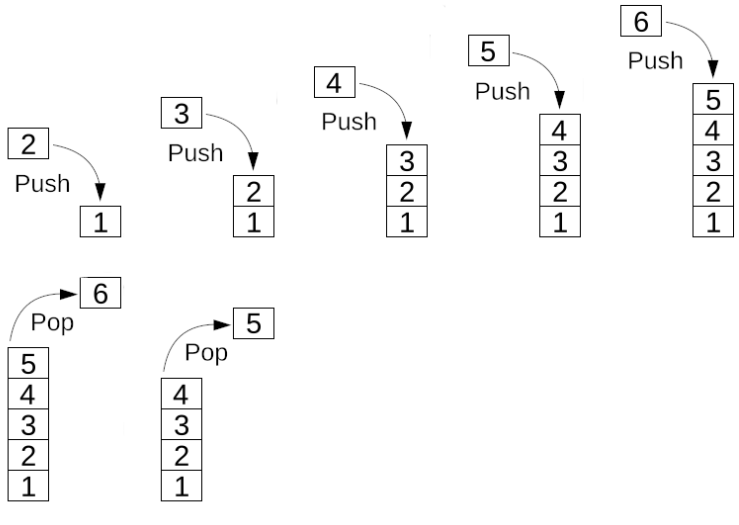
Stack



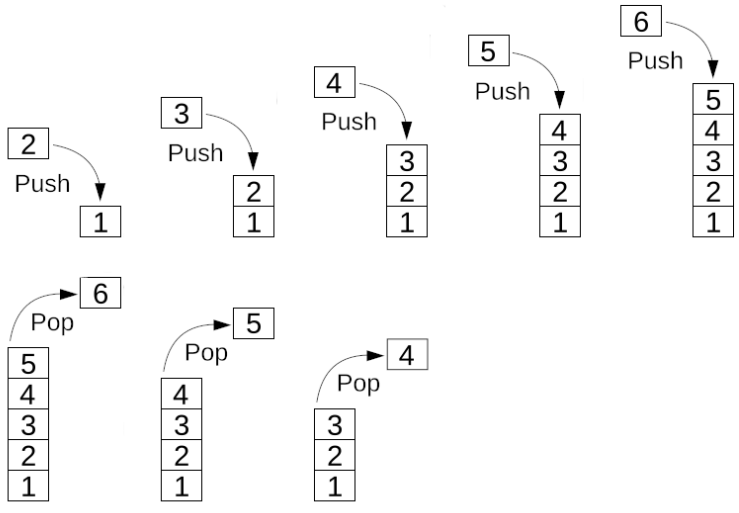
Stack



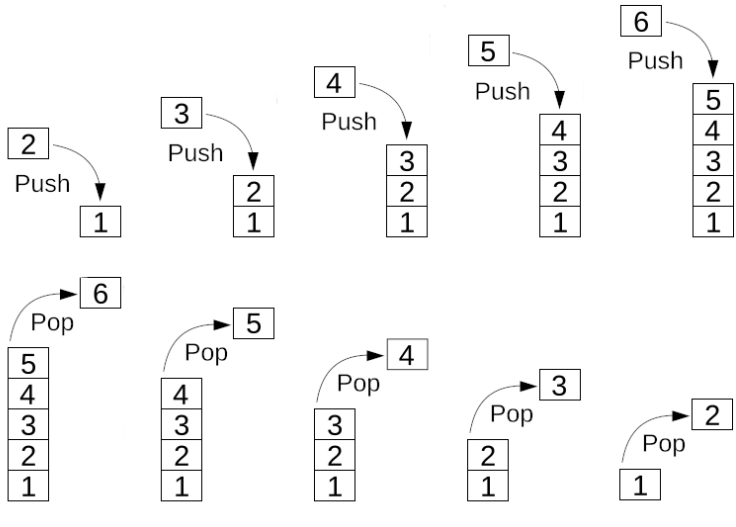
Stack


2
Push
1
3
Push
2
1
4
Push
3
2
1
5
Push
4
3
2
1
6
Push
5
4
3
2
1
6
Pop
5
4
3
2
1
5
Pop
4
3
2
1

Stack


2
Push
1
3
Push
2
1
4
Push
3
2
1
5
Push
4
3
2
1
6
Push
5
4
3
2
1
6
Pop
5
4
3
2
1
5
Pop
4
3
2
1
4
Pop
3
2
1

Stack


2
Push
1
3
Push
2
1
4
Push
3
2
1
5
Push
4
3
2
1
6
Push
5
4
3
2
1
6
Pop
5
4
3
2
1
5
Pop
4
3
2
1
4
Pop
3
2
1
3
Pop
2
1
2
Pop
1

What does this get you?

- You remember DFAs and NFAs???

What does this get you?

- You remember DFAs and NFAs???
- We could remember only the **current state**

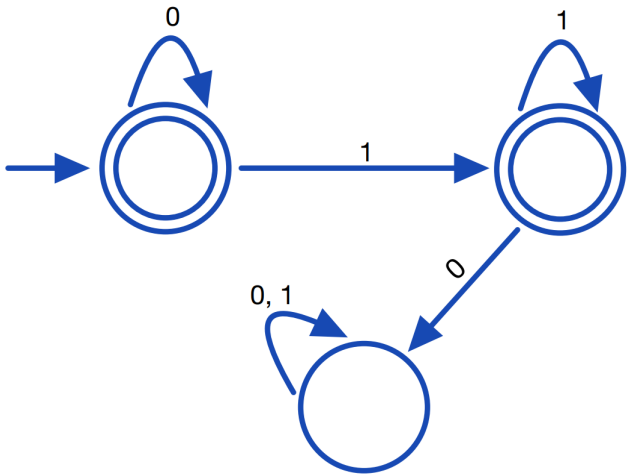
What does this get you?

- You remember DFAs and NFAs???
- We could remember only the **current state**
- The stack gives us a sort of **memory**

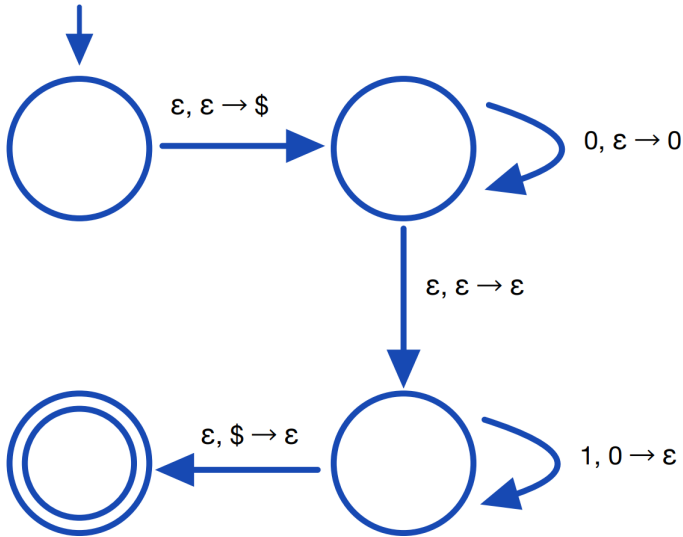
What does this get you?

- You remember DFAs and NFAs???
- We could remember only the **current state**
- The stack gives us a sort of **memory**
- NFA + Stack $\leftrightarrow$ Context Free Grammar

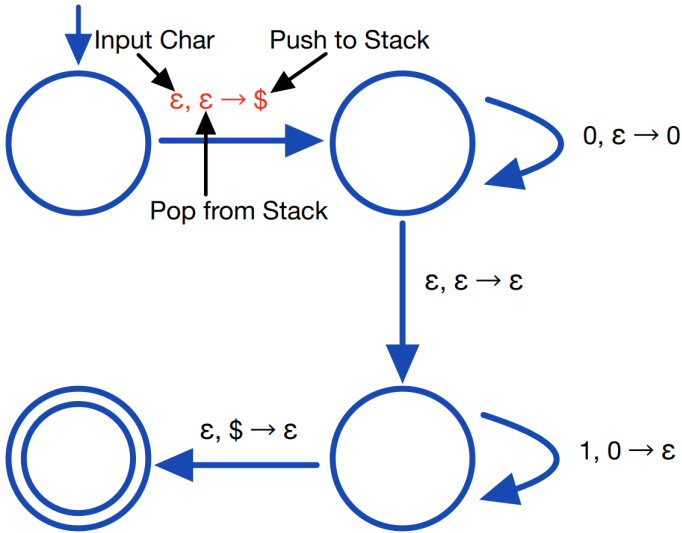
DFA



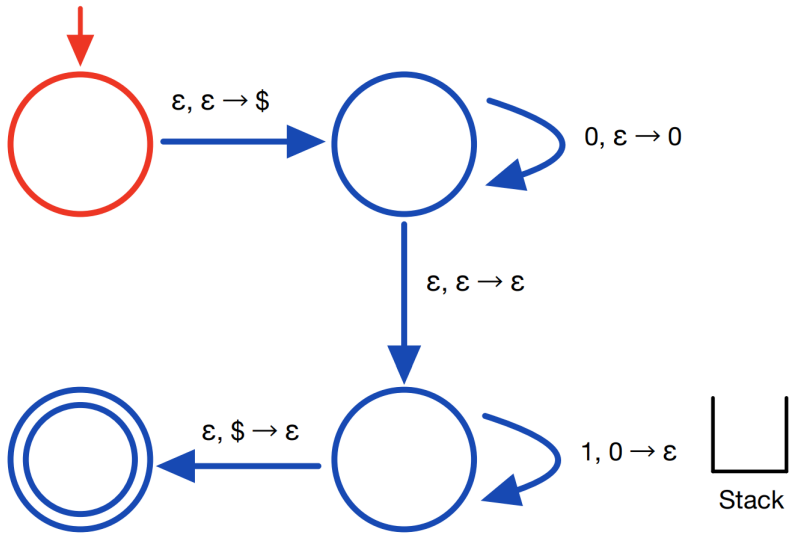
PDA for $\{ 0^n 1^n,\ n \geq 0 \}$



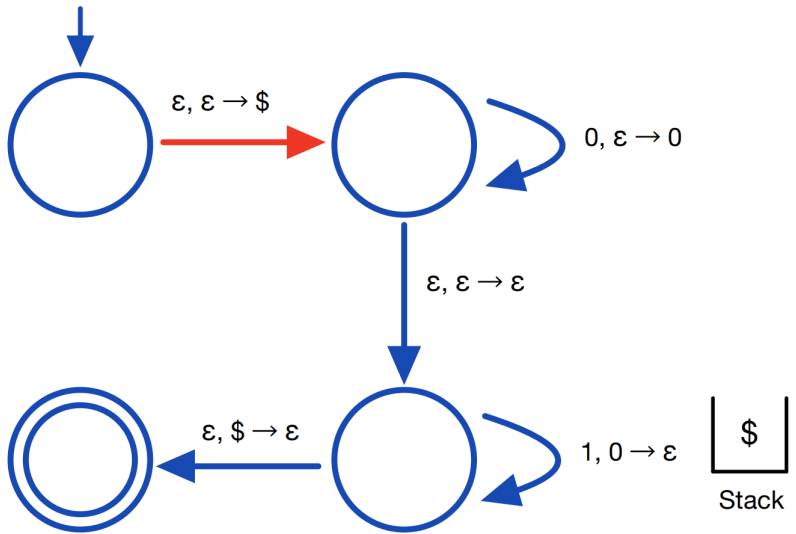
PDA for $\{ 0^n 1^n,\ n \geq 0 \}$



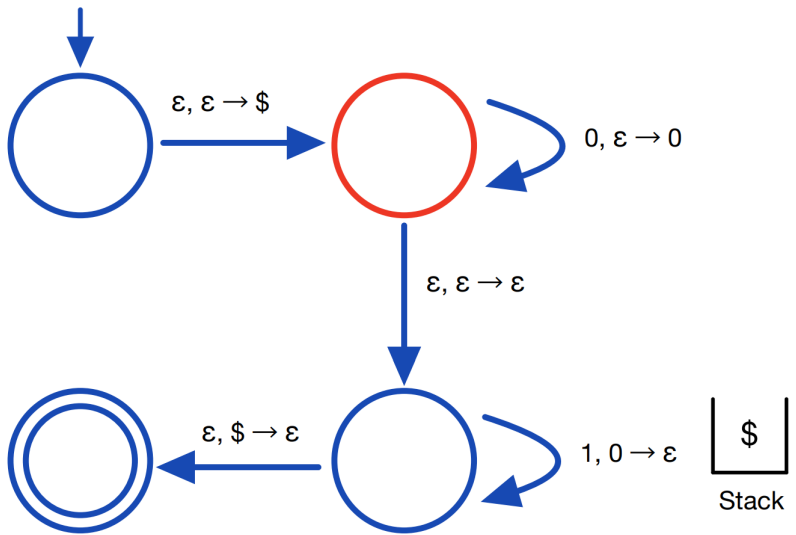
$0011 \stackrel{?}{\in} \{ 0^n 1^n,\ n \geq 0 \}$



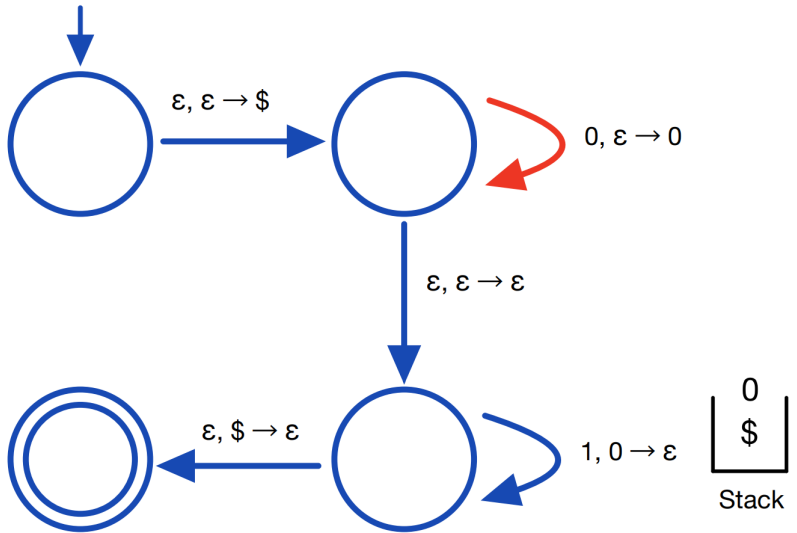
$0011 \stackrel{?}{\in} \{ 0^n 1^n,\ n \geq 0 \}$



$0011 \stackrel{?}{\in} \{ 0^n 1^n,\ n \geq 0 \}$

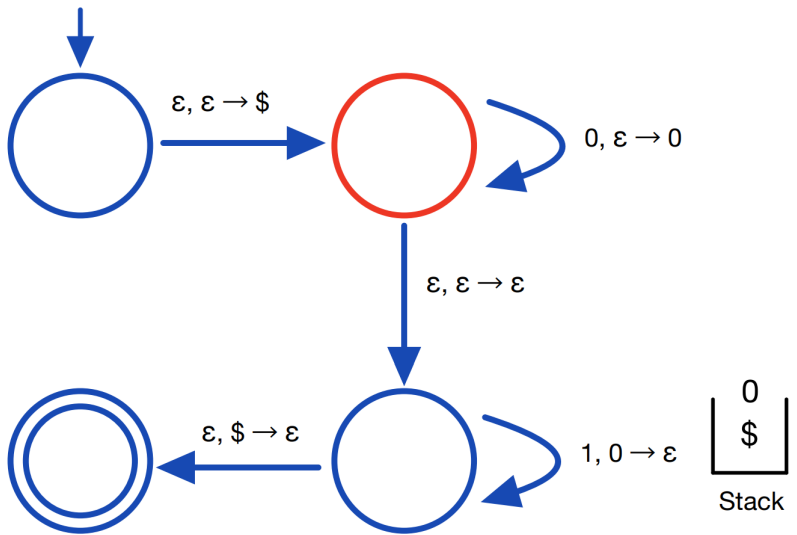


$$0011 \overset{?}{\in} \{\, 0^n 1^n,\ n \geq 0 \,\}$$

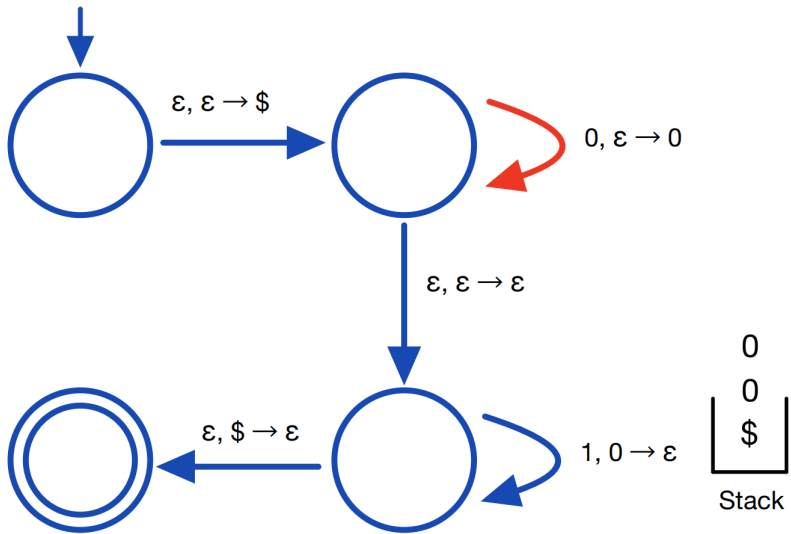


Σ

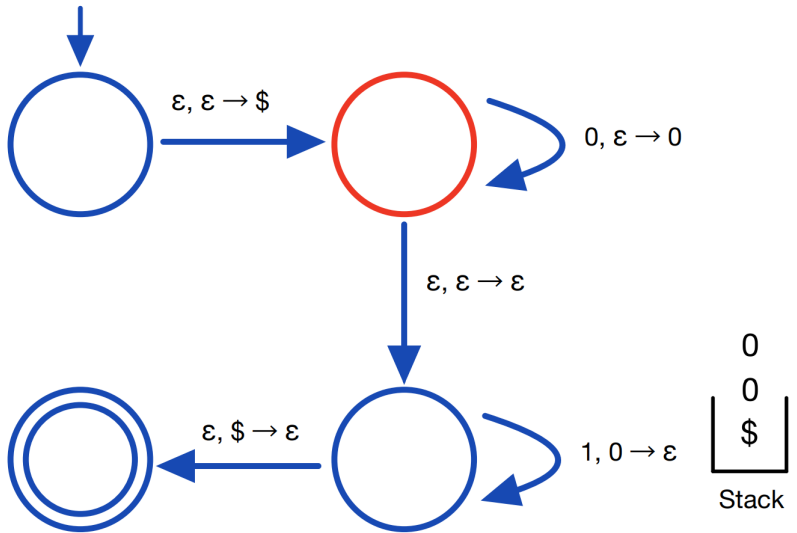
$$0011 \overset{?}{\in} \{\, 0^n 1^n,\ n \geq 0 \,\}$$



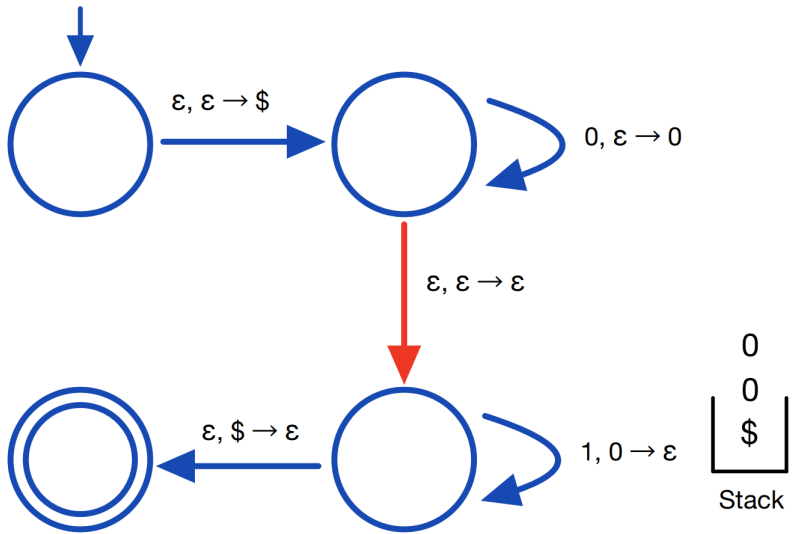
$0011 \stackrel{?}{\in} \{\, 0^n 1^n,\ n \geq 0 \,\}$



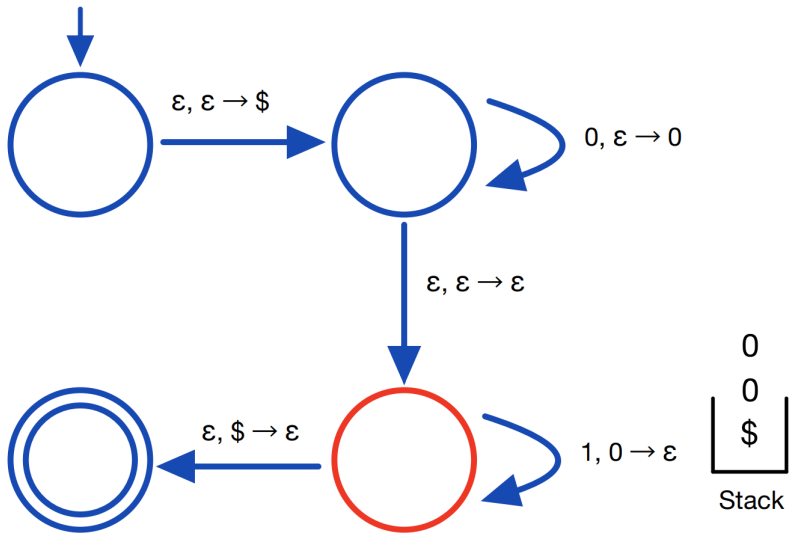
$0011 \stackrel{?}{\in} \{\, 0^n 1^n,\ n \geq 0 \,\}$



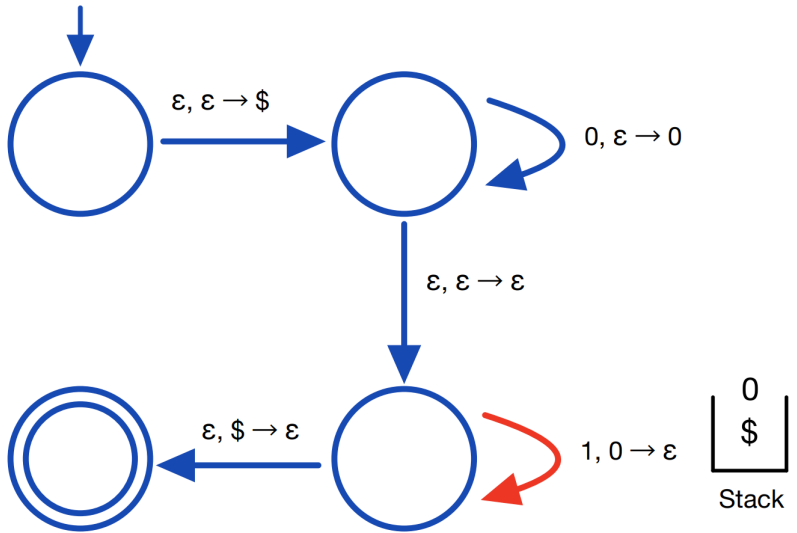
$0011 \stackrel{?}{\in} \{\, 0^n 1^n,\ n \geq 0 \,\}$



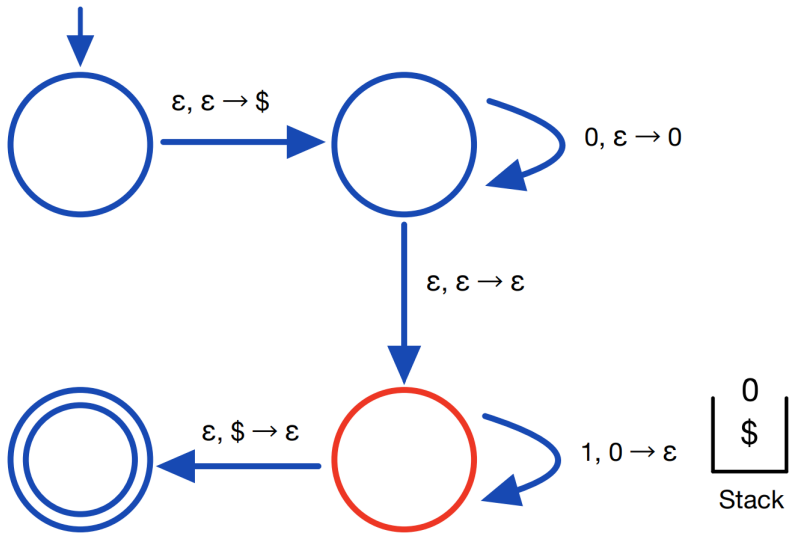
$0011 \stackrel{?}{\in} \{ 0^n 1^n,\ n \geq 0 \}$



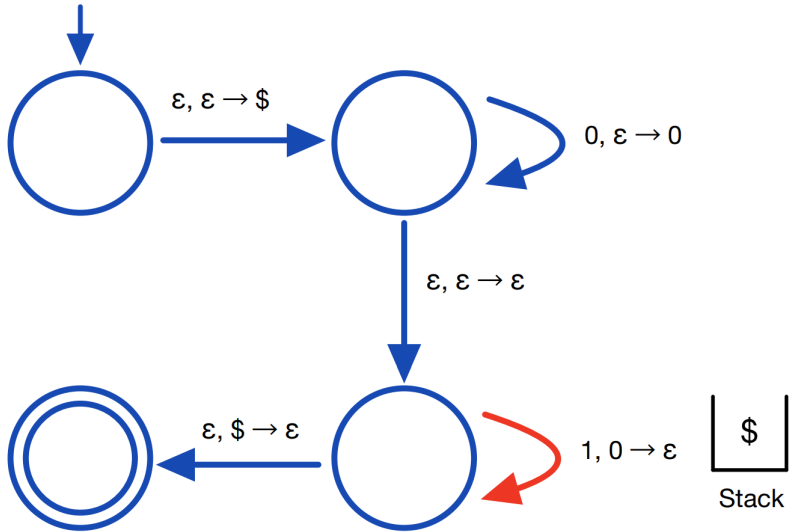
$0011 \stackrel{?}{\in} \{ 0^n 1^n,\ n \geq 0 \}$



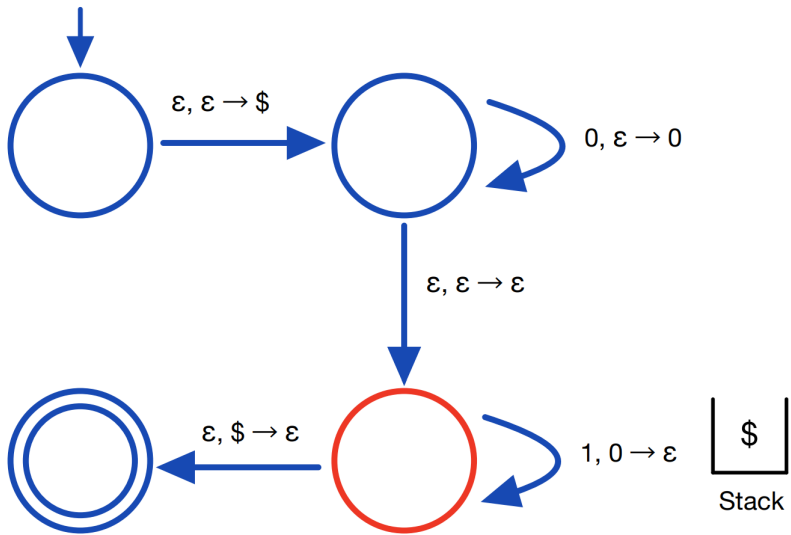
$0011 \stackrel{?}{\in} \{\, 0^n 1^n,\ n \geq 0 \,\}$



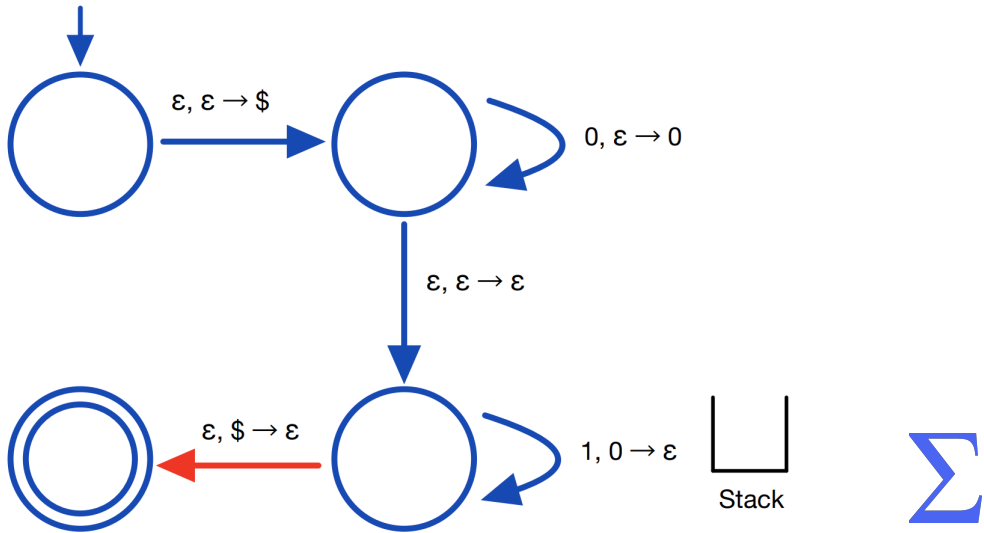
$0011 \stackrel{?}{\in} \{\, 0^n 1^n,\ n \geq 0 \,\}$



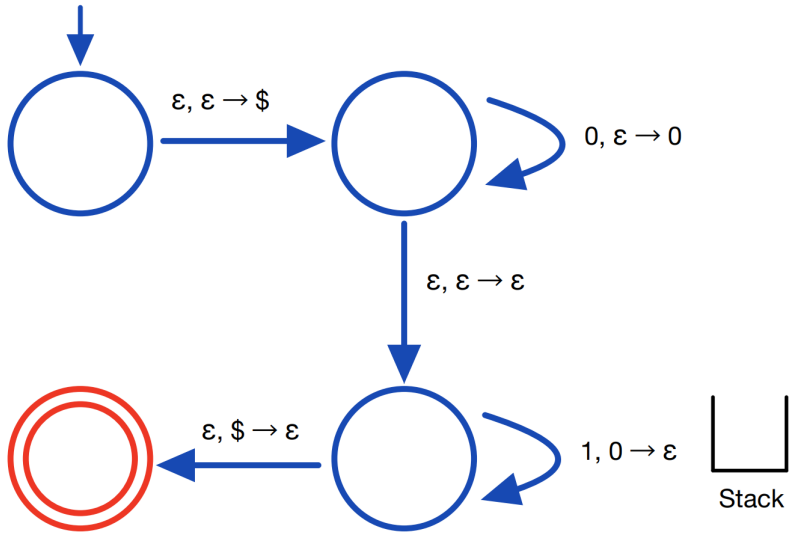
$$0011 \overset{?}{\in} \{\, 0^n 1^n,\ n \geq 0 \,\}$$



$0011 \stackrel{?}{\in} \{\, 0^n 1^n,\ n \geq 0 \,\}$



$0011 \stackrel{?}{\in} \{\, 0^n 1^n,\ n \geq 0 \,\}$



Questions?

Questions!

- Come up with a CFG and a PDA to match the following language

$$\{\, w \mid w \text{ has as many } \texttt{a}\text{'s as } \texttt{b}\text{'s} \,\}$$

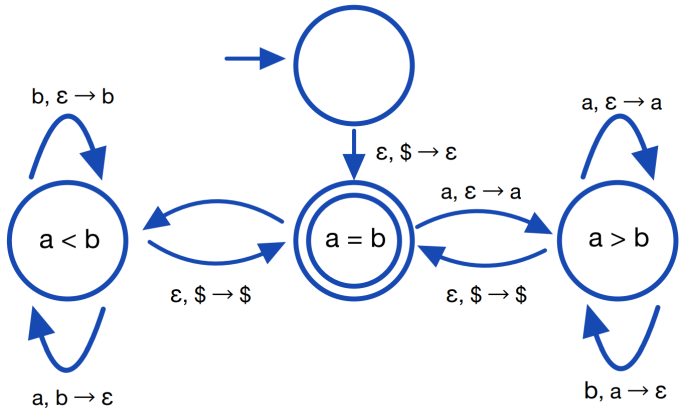
Answers

Come up with a CFG and a PDA to match the following language

$$\{\, w \mid w \text{ has as many } \mathtt{a}\text{'s as } \mathtt{b}\text{'s} \,\}$$

$$S \;\to\; \varepsilon \mid S\mathtt{ab} \mid \mathtt{a}S\mathtt{b} \mid \mathtt{a}S\mathtt{b}S$$

Answers

Come up with a CFG and a PDA to match the following language

$$\{\, w \mid w \text{ has as many } \texttt{a}\text{'s as } \texttt{b}\text{'s} \,\}$$