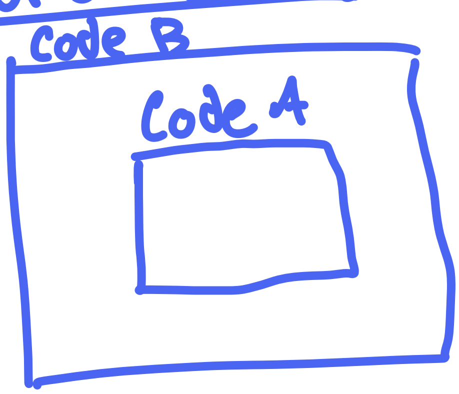
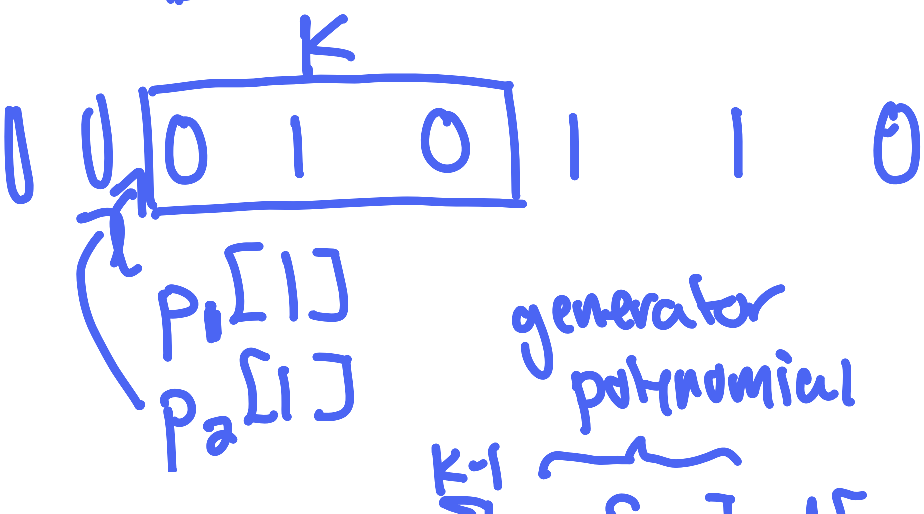
$$\Sigma$$

Turbo Codes!

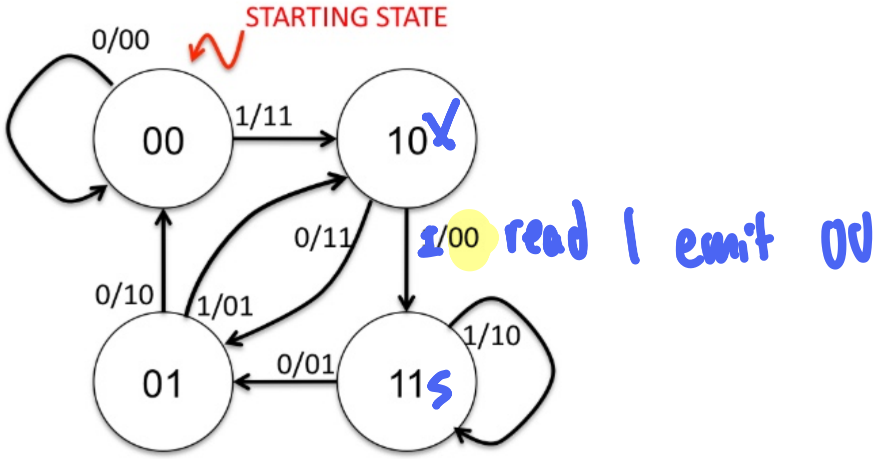


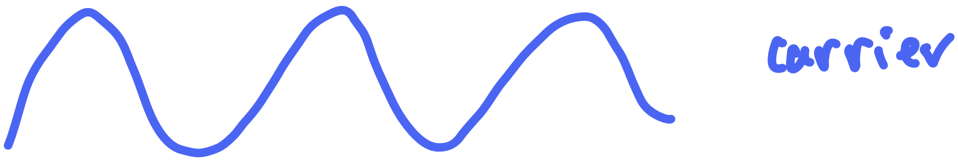


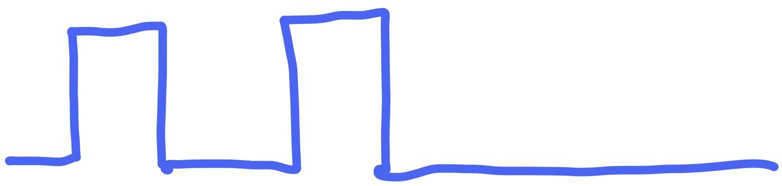
generator polynomial

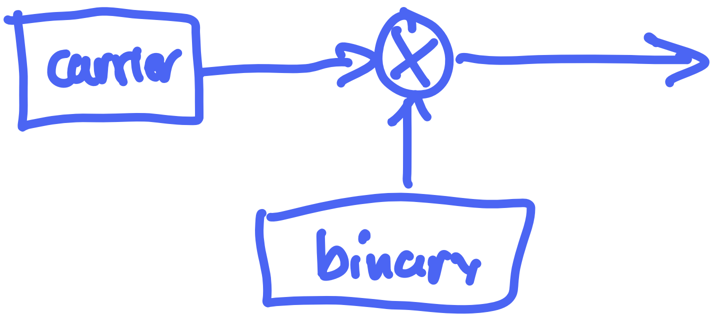
$$p_i[n] = \sum_{j=0}^{K-1} \overbrace{g_i[j]}^{\text{generator polynomial}} \cdot A[n-j] \mod 2$$

$$(1 \quad 1 \quad 0)$$









hard decision = "it is 1" / "it is 0"

soft decision = "it is 1 w/ likelihood..."

Decoding

Probability p of bit errors $\leq 1/2$

$\max_{c} \mathbb{P}\{r \mid c\}$ ← codeword

(r: what we got)

Let $\tilde{c}$ w/ hamming dist d from r

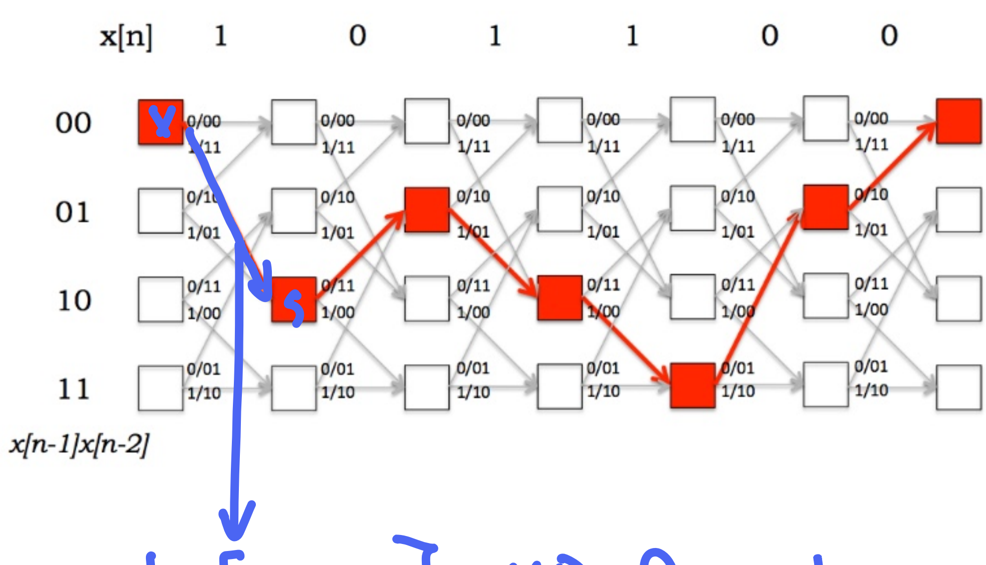
$\mathbb{P}\{r \mid \tilde{c}\} = p^d (1-p)^{N-d}$

$\log \mathbb{P}\{r \mid \tilde{c}\} = d \log p + (N-d)\log(1-p)$

$= d \log \frac{p}{1-p} + N \log(1-p)$ ($\frac{p}{1-p} < 1$)

$\log \mathbb{P}\{r \mid \tilde{c}\}$ is minimized by minimizing d

Goal: find seq of state transitions to min d



$BM[x \to s]$ = HD from branch to actual received bits

Let α, β be legal prev vertices

$$PM[s, i+1] = \min_{x \in \{\alpha, \beta\}} \left(PM[x, i] + BM[x \to s, i] \right)$$

$PM[s, n]$ gives best approx of path for reconstructing codeword

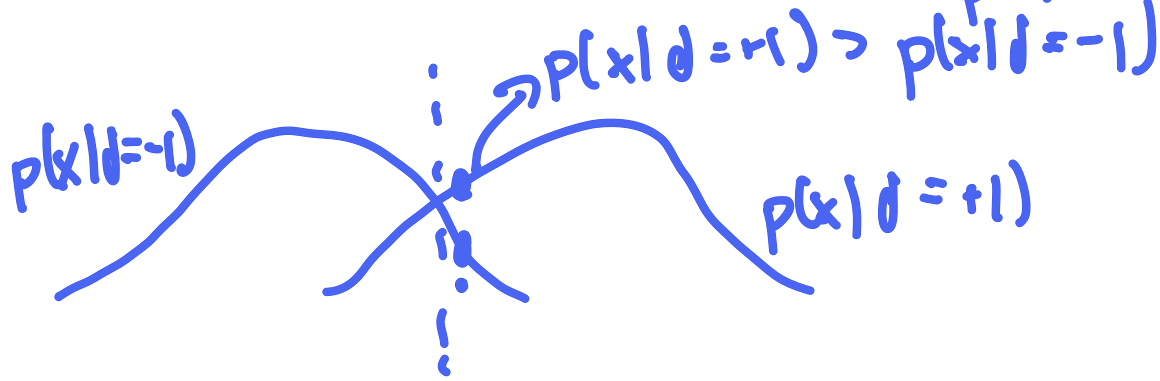
signal noise measurement

Thm $$\mathbb{P}\{d = i \mid x\} = \frac{p(x \mid d=i)\,\mathbb{P}\{d=i\}}{p(x)}$$

data bit belongs to ith signal class (0 or 1)

$$p(x) = \sum_{i=1}^{n} p(x \mid d=i)\,\mathbb{P}\{d=i\}$$

pdf



$$\mathbb{P}\{d = +1 \mid x\} > \mathbb{P}\{d = -1 \mid x\}$$ pick 1, else pick 0

$$\frac{\mathbb{P}\{d=+1\}\,p(x \mid d=+1)}{\mathbb{P}\{d=-1\}\,p(x \mid d=-1)}$$

if > 1 pick +1

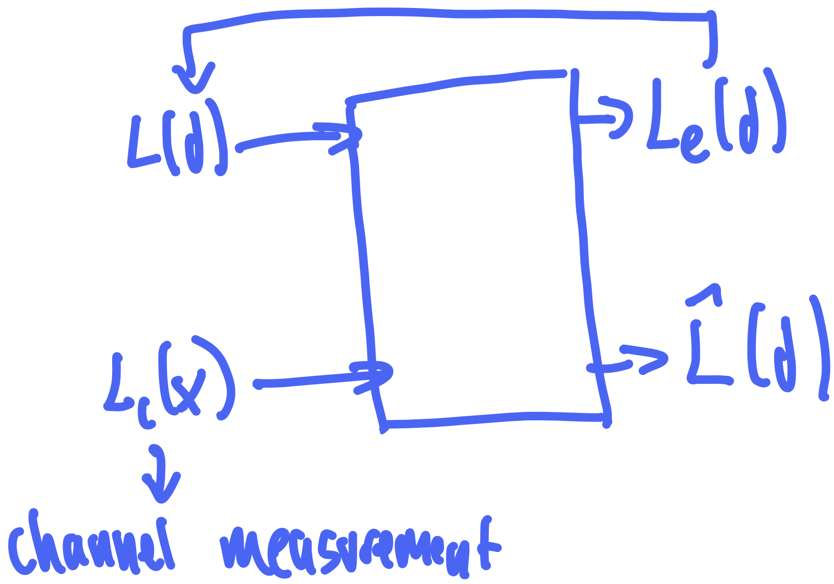
else ret 0

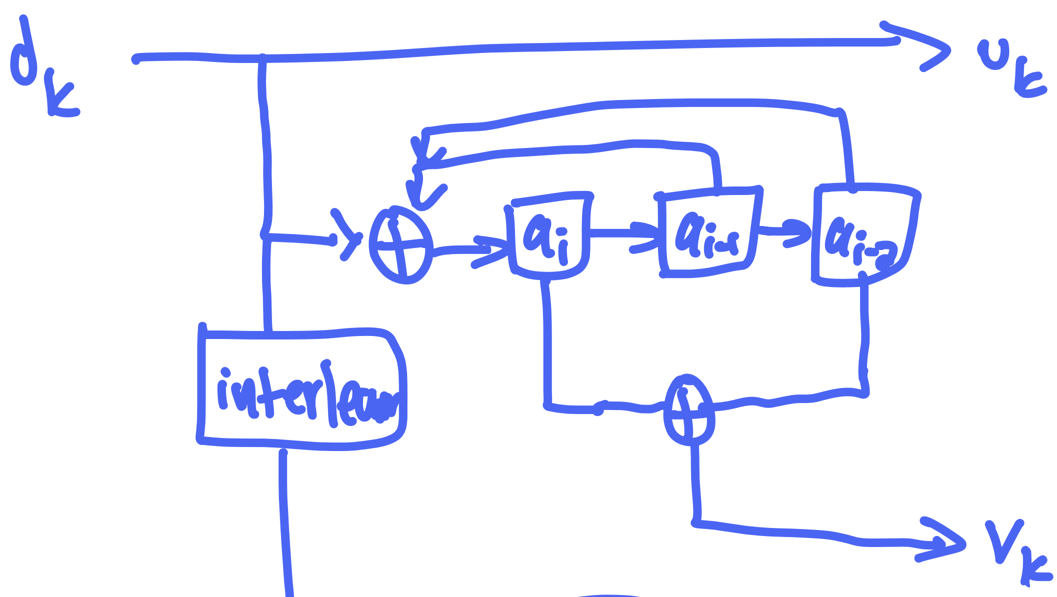
assume 1s and 0s equally likely

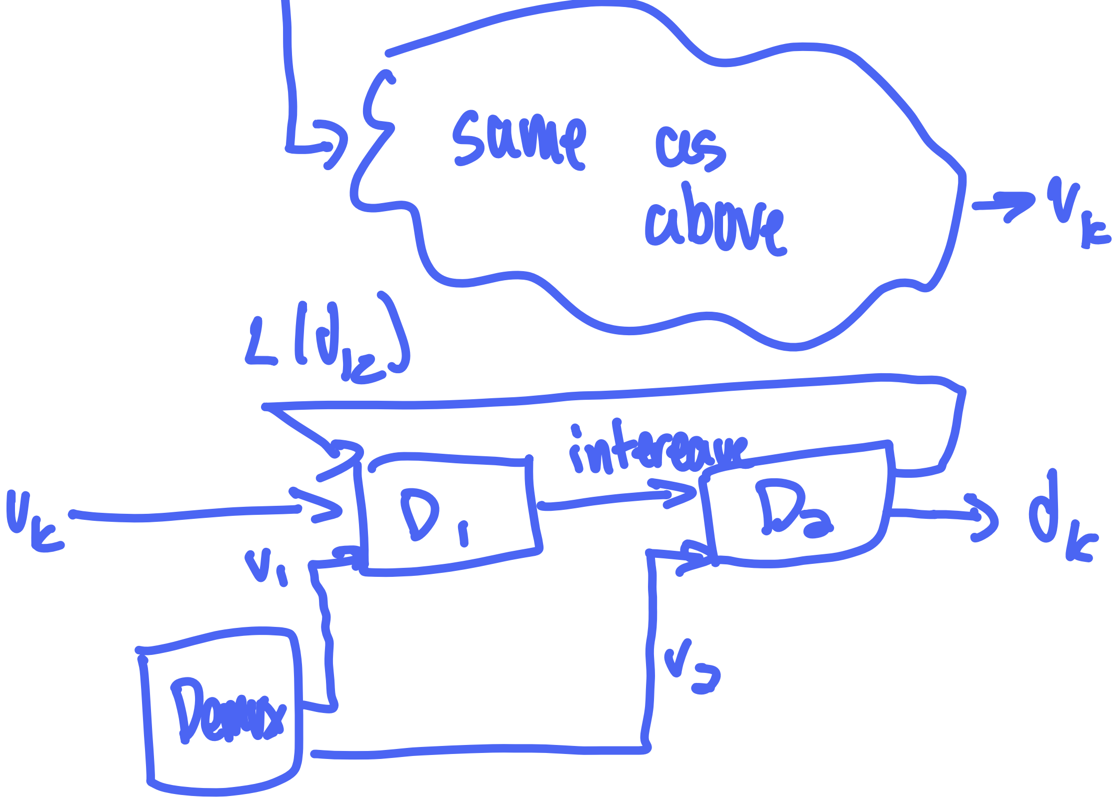
$$\log \frac{\mathbb{P}\{d=+1\}\, p(x \mid d=+1)}{\mathbb{P}\{d=-1\}\, p(x \mid d=-1)}$$

$$= \log\left[\frac{\mathbb{P}\{d=+1\}}{\mathbb{P}\{d=-1\}}\right] + \log\left[\frac{p(x \mid d=+1)}{p(x \mid d=-1)}\right]$$

$$= L(d) + L(x \mid d)$$







10^{-5} bit error prob.