Fast Inverse Square Root

Hassam Uddin

Outline

Representing the Reals

Abusing IEEE-754 for fun and profit

Quake's Fast Inverse-Square-Root

Section 1

Representing the Reals

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In a computer, this has some downsides though.

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None of these are particularly ideal, we are either severely limiting the largest number we can represent, or the smallest magnitude of precision we have.

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- A nice property of binary is that the first bit of a number in this scientific notation will *always* be 1.
- We represent a floating point number x as $\pm q \cdot 2^m$, where m is the exponent, and q is the "significand" of the form $1.f$. We refer to f as the fraction, or mantissa.

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We cannot represent 27, we're stuck approximating it as 28 or 24.

IEEE-754 Single Precision

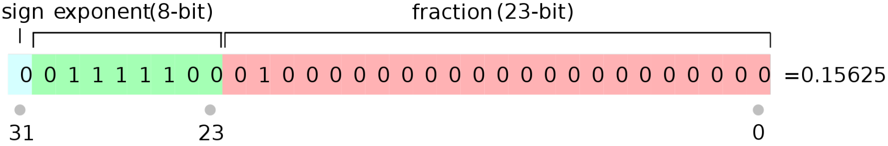
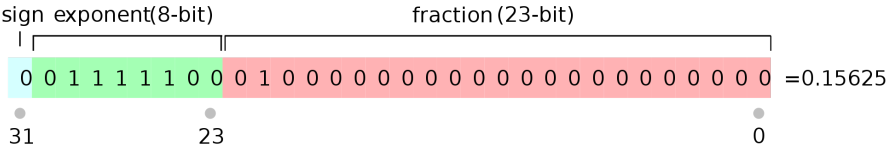


Figure: ~~Stolen~~ Borrowed from CS 357 Notes

IEEE-754 is *very* similar to a floating point representation but with a few tweaks.

$$x = (-1)^s 1.f \cdot 2^m$$

Down to the bits



- We use 1 bit for the sign, s, leaving us 31 bits.
- We use 8 bits for the exponent, giving us 255 possible exponents. We write $m = c - 127$, where c is the actual exponent stored in the binary representation. We also reserve $c = 0$ and $c = 255$ for special cases. The largest exponent is 127 and the smallest exponent is -126.
- The remaining 23 bits are the fractional part, also known as the mantissa.

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- NaN? We set all 1s in the exponent again, but make the mantissa non-zero.

Rounding values is an important consideration in most cases, take CS 357 (or just watch the two lectures associated with floating point numbers) to understand how we use floating points and how we should be careful with them.

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Converting an integer to a float is less clean, so I'll leave that to you.

Section 2

Abusing IEEE-754 for fun and profit

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- Recall that we can approximate $\log(1 + x) \approx \log(x)$, and we can add an error correction factor μ, to make our approximation even tighter.
- So our log is now:

$$\frac{f}{2^{23}} + \mu + c - 127 = \frac{1}{2^{23}}(f + c \cdot 2^{23}) + \mu - 127$$

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- How easy would it be to go from $\log_2(x_{float})$ back to the regular number?
- We just undo the linear transform: we've gotten all the log properties for free!

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- So, we can compute $2^{1/2 \cdot (k_1 x_{int} + k_2)}$
- How do we find our constants k_1, k_2, and do this computation quickly?
- Let's detour into taking the *inverse* square root

Section 3

Quake's Fast Inverse-Square-Root

A detour into history

```c
float q_rsqrt(float number)
{
  long i;
  float x2, y;
  const float threehalfs = 1.5F;

  x2 = number * 0.5F;
  y  = number;
  i  = * ( long * ) &y; // evil floating point bit level hacking
  i  = 0x5f3759df - ( i >> 1 ); // what the fuck?
  y  = * ( float * ) &i;
  y  = y * ( threehalfs - ( x2 * y * y ) ); // 1st iteration
  // y  = y * ( threehalfs - ( x2 * y * y ) ); // 2nd iteration, this can be removed

  return y;
}
```

Modernize

```
constexpr float Q_rsqrt(float number) noexcept
{
  // only allow on IEEE-754 floats
  static_assert(std::numeric_limits<float>::is_iec559);

  // what the fuck? (left for historical accuracy)
  // make use of std::bit_cast to avoid undefined behavior
  float const y = std::bit_cast<float>(
    0x5f3759df - (std::bit_cast<std::uint32_t>(number) >> 1));
  return y * (1.5f - (number * 0.5f * y * y));
}
```

evil floating point bit level hacking

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or

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- Let's solve for the bit representation of Y: $f_Y + c_Y \cdot 2^{23}$:

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- Let's solve for the bit representation of Y: $f_Y + c_Y \cdot 2^{23}$:

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- Historically, it's unknown, and the choice of constant used in Quake is actually not optimal.

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- How did we choose the "magic constant" μ?
- Historically, it's unknown, and the choice of constant used in Quake is actually not optimal.
- If you were doing this in your own program, plot the error and minimize.

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- Are we done?
- We are quite close, but we've introduced a decent amount of error in our assumptions.

Newton's Method, another detour

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  $0 = f(x_0) + f'(x_0)(x_1 - x_0) \implies x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

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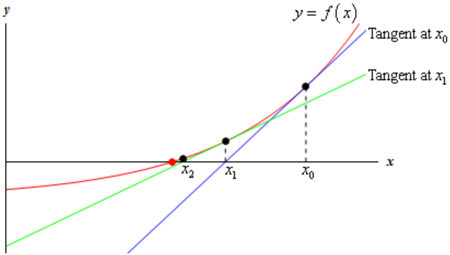

Figure: Paul's Math Notes

Newton's method, on inverse square root

- We want to find $\frac{1}{\sqrt{x}}$, so minimize $\mathrm{error}(y) = \frac{1}{y^2} - x$
- Plugging into Newton's method, we have:

$$y_1 = y_0 - \frac{{y_0}^{-2} - x}{-2{y_0}^{-3}} = \frac{1}{2} y_0 (3 - x y_0^2)$$

Another look

$$\frac{1}{2}y_0(3 - xy_0^2)$$

```cpp
constexpr float Q_rsqrt(float number) noexcept
{
  // only allow on IEEE-754 floats
  static_assert(std::numeric_limits<float>::is_iec559);

  // what the fuck? (left for historical accuracy)
  // make use of std::bit_cast to avoid undefined behavior
  float const y = std::bit_cast<float>(
    0x5f3759df - (std::bit_cast<std::uint32_t>(number) >> 1));
  return y * (1.5f - (number * 0.5f * y * y));
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- The inverse square root does not have any divisions, so it is "fast."
- Quake uses this for the inverse square root because taking the inverse square root of a vector's length is a common operation to normalize a vector.
- We can approximate a *lot* of functions using this approach while avoiding any divisions.

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#include <limits>
#include <cstdint>
#include <cmath>

float Q_rsqrt(float number) noexcept
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  return y * (1.5f - (number * 0.5f * y * y));
}

float inverse_sqrt(float f) {
    return 1 / sqrtf(f);
}
```

x86-64 gcc (trunk) (Editor #1)

x86-64 gcc (trunk) -O2 -ffast-math -std=c++20

A Output... Filter... Libraries Overrides + Add new... Add tool...

```asm
Q_rsqrt(float):
        movd    edx, xmm0
        mov     eax, 1597463007
        mulss   xmm0, DWORD PTR .LC0[rip]
        shr     edx
        sub     eax, edx
        movd    xmm2, eax
        movaps  xmm1, xmm2
        mulss   xmm1, xmm2
        mulss   xmm0, xmm1
        movss   xmm1, DWORD PTR .LC1[rip]
        subss   xmm1, xmm0
        mulss   xmm1, xmm2
        movaps  xmm0, xmm1
        ret
inverse_sqrt(float):
        movaps  xmm1, xmm0
        rsqrtss xmm1, xmm1
        mulss   xmm0, xmm1
        mulss   xmm0, xmm1
        mulss   xmm1, DWORD PTR .LC3[rip]
        addss   xmm0, DWORD PTR .LC2[rip]
        mulss   xmm0, xmm1
        ret
```

All is not lost

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- Inverse square root is such a common operation that it is built into modern hardware
- But, keep in mind, when you're doing any computation, logs and powers are just a cast and linear transformation away.

Questions?

Truth is much too complicated to allow anything but approximations.

— John Von Nuemann (1947)